

A Novel Approach for Analyzing Temporal Influence Dynamics in Social Networks

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Abstract: Features like assortative mixing; high clustering, short average path lengths, broad degree distributions, and community structure have been the subject of numerous recent social network studies. All of these features are met by the model that is introduced in this study. Additionally, our model enables interactions between various communities, fostering a rich network environment. The asymptotically scale-free degree distribution is maintained by our model, which achieves a high clustering coefficient. In our model, the community structure is generated by a mix of mechanisms involving implicit preferential attachment and random attachment. We expand our consideration to include neighbor of neighbor of Initial Contact (NNIC) as well, in contrast to earlier approaches that solely focused on neighbor of Initial Contact (NIC) as an implicit preferential contact. If a newly added vertex chooses more than one initial contact, this extension makes it possible for contacts between those initial contacts to occur. Consequently, our model facilitates the development of complex social networks beyond those used as basic references. Finally, we conduct centrality calculations on both the existing model and our developed model, providing a comparative analysis of the results.

Keywords: Social networks; initial contact; secondary contact; tertiary contact; clustering; centrality.

1 Introduction

In contemporary research practices, there's a notable trend towards interdisciplinary collaboration, where experts from various fields come together to tackle complex problems or explore new frontiers. This trend is exemplified by instances such as stock market analysts seeking insights from physics simulations to improve their predictive models. This cross-pollination of ideas and methodologies highlights the increasing recognition of the value in looking beyond traditional disciplinary boundaries to find innovative solutions [1].

This shift towards collaboration across domains reflects a broader societal recognition that many of today's most pressing challenges are multifaceted and cannot be adequately addressed within the confines of a single discipline. Just as social networks consist of diverse communities with their own distinct characteristics and interactions, so too does the realm of academic research comprise diverse fields, each with its own methodologies, theories, and expertise [2].

Recognizing the potential benefits of collaboration across these different academic

communities, our research endeavors to develop a novel model that facilitates such interdisciplinary interactions within academic settings. This model is designed to enable individuals from disparate fields to come together, share insights, and collaborate on projects while still preserving the unique characteristics and structures of their respective academic communities.

At the heart of this model lies an understanding of social networks as networks of interconnected nodes, where each node represents an individual or organization, and the connections between them represent various types of relationships or interactions. By leveraging this framework, our model seeks to create pathways for collaboration that transcend disciplinary boundaries, ultimately fostering greater innovation and knowledge exchange within the academic community.

For decades, social scientists have studied social networks in depth [3-5] to learn about both the micro phenomena, like how networks form and evolve, and the macro processes, like the dissemination of information, the propagation of disease, the spread of rumors, the exchange of ideas, etc. Researchers have looked at many different kinds of social networks, including those that facilitate professional collaboration [6-8], online dating [9], and the process by which individuals form their opinions. Financial, cultural, educational, familial, relational, and other social networks are all a part of this. Sociology, elementary mathematics, and graph theory are all parts of social networks, which are able to form relationships between nodes. A social network's most basic mathematical component is a graph. Hierarchical community structure [10], the small world property [11], and the power law distribution of nodes degree are all significant social network properties, with the Barabasi-Albert model of scale-free networks being the most basic [12]. There is a growing interest among scientists in the new areas of study made possible by the proliferation of online social networks. One of the most well-known online social networks is Facebook, where users can make new friends, communicate with existing ones, and share information about themselves through profile updates. The presence of community structure, short average path lengths, assortative mixing [13-15], and broad degree distributions are supposedly essential features of social networks. In a growing community, the intercommunity connections are relatively sparse, and the set of vertices has dense internal connections. Previously, only friend information was updated in the old model [16-19], but in the newer model, information about friends of friends is also updated. Data and information sharing will be lightning fast, even though this model builds a complicated social network. By preserving the current community structure, this efficiently achieves the true goal of social networking while allowing for faster growth. Here we have also performed centrality calculations on both the existing and newly developed models. Centrality is the fundamental property used to study the topology of network flows and information broadcast speed [20-22]. This model works for social and biological graphs.

2 Novel Network Growth Algorithm

The algorithm is composed of three distinct processes, each serving a crucial role in generating the graph:

2.1 Random Attachment

In this process, nodes are added to the network randomly, without any preference or bias towards existing nodes. This mimics the spontaneous formation of connections that can occur in real-world social networks, where individuals may establish relationships with others regardless

of their existing connections or characteristics.

2.2 Implicit Preferential Contact with Neighbors of Initial Contact

This step introduces a preference for nodes that are connected to the initial contact. When a new node joins the network, it is more likely to establish connections with nodes that are already connected to the initial contact. This reflects the tendency observed in social networks, where individuals are often introduced to new contacts through their existing connections, leading to the formation of clusters or communities within the network.

2.3 Contact with Neighbor of Neighbor (Tertiary)

Building upon the concept of preferential attachment, this extension proposes a connection between the initial contact and its Neighbor of Neighbor, or tertiary contact. By facilitating connections between nodes that are two degrees of separation away from the initial contact, this process promotes the expansion of the network beyond immediate neighbors, fostering a more interconnected and expansive structure.

By combining these processes, the algorithm generates a graph that exhibits characteristics similar to real-world social networks, such as clustering, assortativity, and the presence of communities. This approach not only captures the inherent complexities of social interactions but also provides insights into the mechanisms driving the formation and evolution of social networks. Figure 1 shows a 40-vertex social network diagram

The algorithm of the model is taken from [1] and generated this graph.

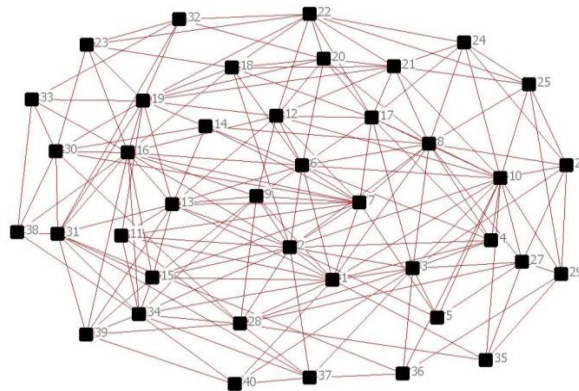


Figure 1: a 40-vertex social network diagram

3 Vertex Degree Distribution

We obtain a close approximation for the vertex degree distribution for a network model that incorporates random initial contact, neighbor of neighbor initial contact, and neighbor of initial contacts into its growth. Exponent $2 < \gamma < \infty$ has been used to derive the power law degree distribution with $p(k) \sim k^{-\gamma}$ [17, 19]. The earlier model's lower bound to the degree exponent γ was also determined to be 3, and this model confirms that.

We build the rate equation that describes the average change in vertex degree during one time step of the network growth. Three procedures contribute to the increase of vertex v_i :

- On average, there will be $\sim t$ vertices whenever a new vertex directly links to v_i at any given time t . In this case, we are picking m_r from among them with a

probability of m_r divided by t .

- Preferred attachment results from selection when a vertex establishes a secondary contact with v_i . A total of $m_r \cdot m_s$ will be involved.
- The tertiary contact between vertices and v_i is also a case of random preferential attachment. This will amount to two milliseconds.

These three processes lead to following rate equation for the degree of vertex v_i [1].

$$\frac{\partial k_i}{\partial t} = \frac{1}{t} \left(m_r + \frac{m_r m_s + 2m_r m_s m_t}{2(m_r + m_r m_s + 2m_r m_s m_t)} k_i \right) \quad (1)$$

From this we get the probability density distribution for degree k_i as

$$P(k) = A B A (k + C)^{-2} m + 2m m^{-3} \quad (2)$$

A , B , and C are the same as before. The distribution changes to a power law $p(k) \sim k^{-\gamma}$ in the limit of large k , where $\gamma = 3 + 2/m_s$ and $m_s > 0$, and $3 < \gamma < \infty$. Therefore, 3 is the minimum value for the degree exponent. But this is irrelevant because the lower bound for the degree exponent is the same as in the old model. The new model's probability density distribution is larger than the old one because the first term of degree exponent's denominator is bigger.

4 Clustering

The rate equation method can also be used to find the clustering coefficient on vertex degree [18]. Here we will look at the time-dependent variation of the number of triangles E_i . There are primarily three processes that produce the triangle surrounding v_i .

The rate equation [1] describes these three processes.

- Vertices the new vertex links to some of its neighbors as secondary contacts, resulting in the formation of a triangle, and v_i is chosen as one of the initial contacts with a probability of m_r/t .
- Triangles are created when a new vertex links to an existing one, either as a primary or tertiary contact, using the selected vertex v_i as a secondary contact.
- The tertiary contact is selected as vertex v_i , and the new vertex links to it as either its primary or secondary contact, resulting in the formation of triangles.

These three process are described by the rate equation [1]

$$\frac{E_i}{t} = \frac{K_i}{t} - \frac{1}{t} \left(m_r - m_r m_s - 3m_r m_s m_t - \frac{5m_r m_s m_t}{2(m_r + m_r m_s + 2m_r m_s m_t)} k_i \right) \quad (3)$$

We arrive the clustering the coefficient

$$c_i(k_i) = \frac{2E_i(k_i)}{k_i(k_i - 1)} \quad (4)$$

Where

$$E_i(k_i) = D \left(k_i \ln(k_i + c) + k_i(F + Gk_i) - Dk_i \ln B + H \ln(k_i + c) + k_i(F + Gk_i) - Dk_i \ln B + H \ln(k_i + c) \right) I \ln(F + Gk_i) + J$$

Where

$$a = m_r m_s + 3m_r m_s m_t - m_r$$

$$b = \frac{5m_r m_s m_t}{2(m_r + m_r m_s + 2m_r m_s m_t)}$$

$$F = \frac{a}{a + b k_{init}}, D = Ab, G = \frac{b}{a + b k_{init}}, H = aA$$

$$I = \frac{a}{b}, J = E_{init} - H \ln B$$

Thus, the clustering coefficient is k -dependent as $c(k) \sim \ln k/k$ for big values of degree k . In contrast to the previous study, which had a clustering coefficient of $c(k) \sim 1/k$, this one has a significantly larger value.

5 Centrality Measure

In the field of social network analysis, centrality measure has received a great deal of attention. Degree centrality, closeness centrality, and betweenness centrality are among the many outcomes that have been quantified [20]. Undirected graphs $G(V, E)$ are the building blocks of any network on which we could potentially compute centrality measures. Here, V stands for a collection of nodes, vertices, points, or actors, and E is the collection of ties or lines that link them [21–24]. Here we can see the distribution of centrality on 40 nodes in a graph. Nodes 2 and 10 are the most central, as is evident. What this means is that the information sent to that node will travel through the network more quickly than to other nodes. Figure 3 showing Centrality in social network graph with 40 vertices. Here bigger the size of the rectangle on each node more is the centrality.

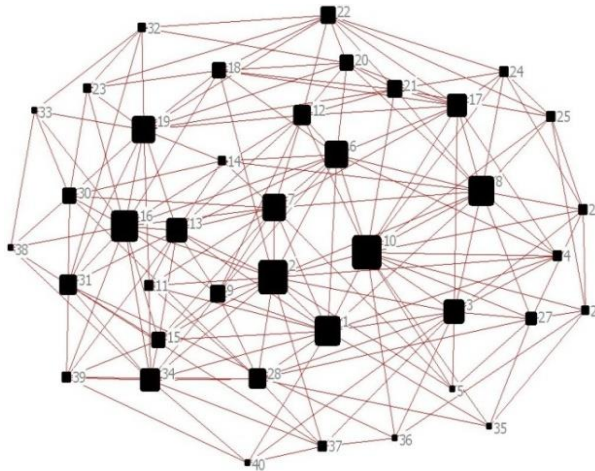


Figure 3: Centrality in social network graph with 40 vertices.

6 Comparison

We have computed the edge-to-vertex and triangle-to-vertex ratios for 40 vertices in our model and presented them here for comparison. You can observe the results in Table 1 shows there was a significant rise in secondary contacts. Our model now includes tertiary contacts as

well, allowing for more complicated and rapid network expansion.

Table 1: Shows the vertices and triangles

| Data on our model | Initial Contact(IC) | Secondary Contacts(SC) | Neighbour of NeighbouIC (NNIC) |
|-------------------|---------------------|------------------------|--------------------------------|
| Vertices | 2.35 | 3.47 | 2.95 |
| Triangles | 0.28 | 6.54 | 6.35 |

Table 2 provides a comprehensive overview of the centrality calculation results for both the existing model and the proposed model. It is worth noting that the introduction of the Neighbor of Neighbor of Initial Contact (NNIC) mechanism leads to a remarkable enhancement in various centrality measures, including degree centrality, closeness centrality, and betweenness centrality.

When comparing the centrality values listed in appendices 1 and 2 for the current and newly developed models, one can discern a significant difference. Specifically, the centrality values for nodes exhibit notable increases when NNIC is integrated into the model. This indicates that NNIC plays a crucial role in augmenting the centrality of nodes within the network.

Furthermore, the introduction of NNIC introduces the possibility of altering the node with the highest centrality. In other words, the node that occupies the top position in terms of centrality may change when NNIC is employed. This underscores the substantial impact that incorporating NNIC can have on the overall network structure and the importance of individual nodes within it.

Overall, the centrality calculation results underscore the effectiveness of NNIC in enhancing the centrality of nodes and potentially reshaping the network's topology and dynamics.

Table 2: Data from existing model run with

| | | Degree | Clo se ness | Between ness |
|----|-------------|---------|-------------------|-----------------|
| 1 | Mean | 24.4 | 50.8 | 2.5 |
| 2 | Std Dev | 9.4 | 5.3 | 2.4 |
| 3 | Sum | 979.4 | 2034 .5 | 103.9 |
| 4 | Variance | 88.4 | 28.1 | 6.2 |
| 5 | SSQ | 27521.3 | 1046 14. | 519.2 |
| 6 | MCSSQ | 3536.4 | 1124 .9 | 249.2 |
| 7 | Euc Norm | 165.8 | 323. 4 | 22.7 |
| 8 | Minimu m | 12.8 | 42.8 | 0.08 |
| 9 | Maximu m | 43.5 | 62.9 | 9.7 |
| 10 | N of Obs | 40.0 | 40.0 | 40.0 |

7 Simulation Results

We have projected the simulation results for a network with 30 vertices, taking into account the edge-to-vertex ratio and the triangle-to-vertex ratio for all 30 nodes. Therefore, the

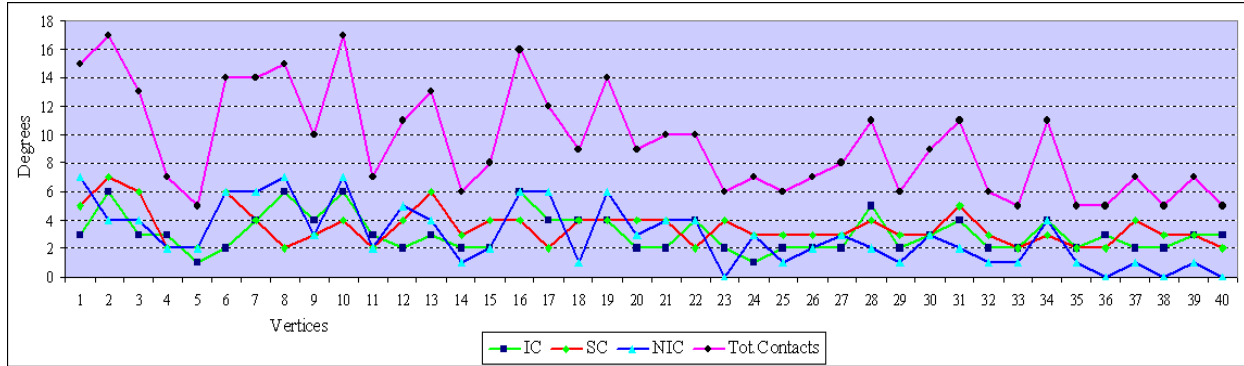


Figure 4: Comparison of results of growing network community: initial contacts are At a much slower rate than secondary contact, initial contact is represented by ■, secondary contacts by ◆, the neighbor of the neighbor of the initial contact by connects to the vertex vi, and the degree of each vertex is indicated by ● when all three types of contacts—initial, secondary, and tertiary—connect to the same vertex vi. In comparison to the current model, the vertices simulation results from Table 1 show that our network community is expanding at a rapid and complicated rate.

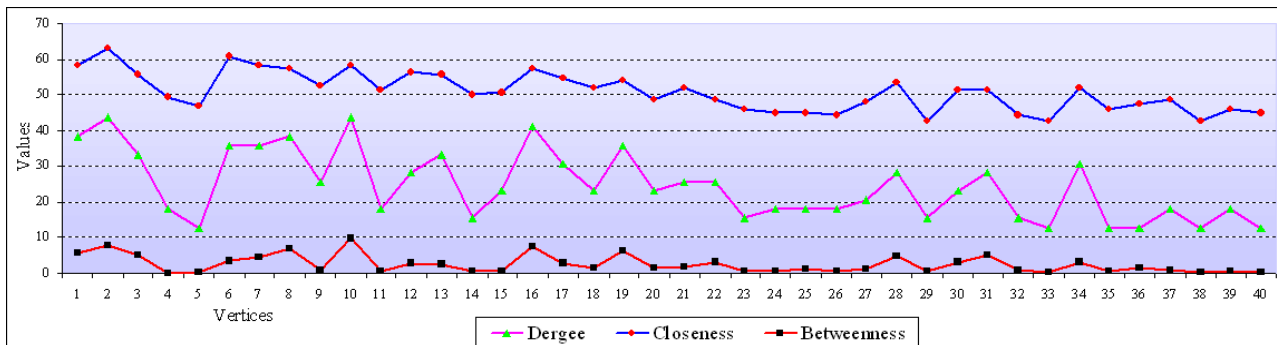


Figure 5: Comparison of results of growing network community of ▲ indicates Degree, ◆ indicates Closeness and ■ indicates Betweenness:

8 Conclusion

Developed in this paper is a model that, when compared to actual social networks, replicates them with remarkable efficiency. In this case as well, the minimum acceptable degree exponent remains unchanged. Consistent with the previous finding for $mt=0$, the probability distribution holds for degrees k . For big values of k , the clustering coefficient's growth rate of $\ln(ki)/ki$ is much higher than the previous result of $1/k$. In the context of academic groups, this is highly beneficial, as it facilitates the rapid exchange of information and the tremendous expansion of research. Consequently, this study presents the results of an efficient but intricate social network

model that, while maintaining the community structure, yields a significant improvement in centrality, edge-to-vertex ratio, clustering coefficient, and probability distribution. Using this model, different research groups can create a new kind of social network that facilitates the rapid dissemination of information, which is crucial for accelerating learning and technological advancement.

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